



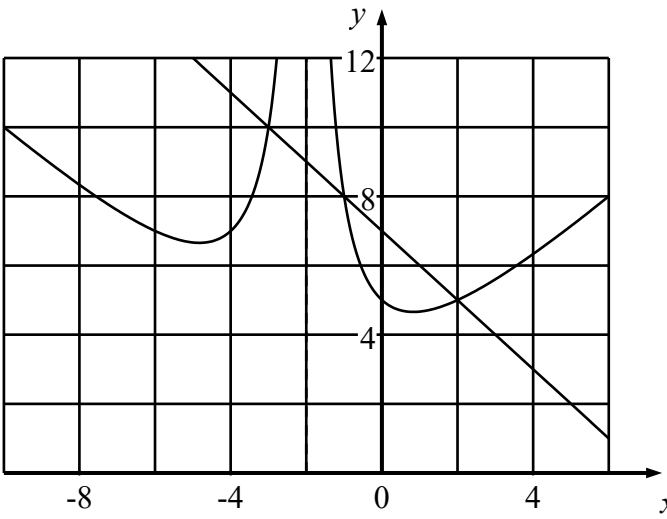
Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2
(WFM02/01)

Question Number	Scheme	Marks
1(a)	$\frac{d^3 y}{dx^3} + 3 \frac{dy}{dx} + 3x \frac{d^2 y}{dx^2} = -2 \sin x$ $\frac{d^3 y}{dx^3} = -2 \sin x - 3 \frac{dy}{dx} - 3x \frac{d^2 y}{dx^2}$	M1M1 A1 (3)
(b)	$\frac{d^3 y}{dx^3} = -3 \times 5 = -15$	B1 (1)
(c)	$\frac{d^2 y}{dx^2} = -3 \times 0 \times 5 + 2 = 2$ $y = 2 + 5x + x^2 - \frac{5}{2}x^3$	B1 M1A1 (3)
[7]		
(a) M1	Accept the dashed notation throughout this question.	
M1	Differentiate $3x \frac{dy}{dx}$ with respect to x . The product rule must be used for $x \frac{dy}{dx}$ with at least one term correct	
M1	Differentiate $\frac{d^2 y}{dx^2}$ and $2 \cos x$. $\frac{d^2 y}{dx^2} \rightarrow \frac{d^3 y}{dx^3}$ $2 \cos x \rightarrow \pm 2 \sin x$	
A1	$\frac{d^3 y}{dx^3} = -3 \left(x \frac{d^2 y}{dx^2} + \frac{dy}{dx} \right) - 2 \sin x$. Give A0 if not rearranged to have $\frac{d^3 y}{dx^3} = \dots$	
(b) B1	$\frac{d^3 y}{dx^3} = -15$ provided 3 terms in result in (a)	
(c) B1	$\frac{d^2 y}{dx^2} = 2$ can be implied by a correct x^2 term in the expansion	
M1	Use of a correct Taylor expansion with their values for $\frac{d^3 y}{dx^3}$ and $\frac{d^2 y}{dx^2}$ 2! or 2, 3! or 6.	
A1	$y = 2 + 5x + x^2 - \frac{5}{2}x^3$ Must include $y = \dots$ or $f(x) = \dots$ provided $f(x)$ has been defined to be y somewhere in the work.	

Question Number	Scheme	Marks
<p>2 (a)</p> $\frac{3r+1}{r(r-1)(r+1)} = \frac{A}{r} + \frac{B}{r-1} + \frac{C}{r+1}$ $\frac{3r+1}{r(r-1)(r+1)} = -\frac{1}{r} + \frac{2}{r-1} - \frac{1}{r+1}$ <p>(b)</p> $\frac{2}{1} - \frac{1}{2} - \frac{1}{3}$ $\frac{2}{2} - \frac{1}{3} - \frac{1}{4}$ $\frac{2}{3} - \frac{1}{4} - \frac{1}{5}$ $\frac{2}{4} - \frac{1}{5} - \frac{1}{6}$ $= 2 - \frac{1}{2} + \frac{2}{2} - \frac{1}{n} - \frac{1}{n} - \frac{1}{n+1}$ $\frac{5}{2} - \frac{2}{n} - \frac{1}{n+1} = \frac{5n(n+1) - 4(n+1) - 2n}{2n(n+1)}, = \frac{5n^2 - n - 4}{2n(n+1)}$ <p>(c)</p> $\sum_2^{20} - \sum_2^{14}$ $= \frac{5 \times 20^2 - 20 - 4}{2 \times 20 \times 21} - \frac{5 \times 14^2 - 14 - 4}{2 \times 14 \times 15}$ $= \frac{13}{210}$	<p>M1A1 (2)</p> <p>M1</p> <p>dM1A1</p> <p>M1, A1 cso (5)</p> <p>M1</p> <p>A1 (2)</p> <p>[9]</p>	
<p>(a)</p> <p>M1</p> <p>A1</p> <p>(b)</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>M1</p> <p>A1cso</p> <p>(c)</p> <p>M1</p> <p>A1</p>	<p>Correct method for obtaining the PFs</p> <p>Correct PFs</p> <p>Show sufficient terms at both ends (eg 3 at start and 2 at end) to demonstrate the cancelling. (This can be implied by correct work at the next line)</p> <p>Must be using PFs of the correct form and start at $r = 2$ unless extra terms are ignored at next stage. Can be split into $\sum \left(\frac{1}{r-1} - \frac{1}{r} \right) + \sum \left(\frac{1}{r-1} - \frac{1}{r+1} \right)$</p> <p>Extract the non-cancelled terms (min 4 correct terms but 5/2 counts as 3 correct)</p> <p>Depends on first M of (b)</p> <p>Correct terms extracted</p> <p>Write terms using the common denominator, numerator need not be simplified. Must start with a min of 3 terms inc terms with denominators n and $(n+1)$</p> <p>Correct answer from correct working</p> <p>Form and use the difference of the 2 summations shown using their result from (b) or an earlier form seen in (b)</p> <p>Correct exact answer, as shown or equivalent</p>	

Question Number	Scheme	Marks
3	 $\frac{x^2 + 3x + 10}{x + 2} = 7 - x$ $x^2 + 3x + 10 = 14 + 5x - x^2$ $x^2 - x - 2 = 0 \quad (x - 2)(x + 1) = 0$ <p>CVs 2, -1</p> $\frac{-(x^2 + 3x + 10)}{x + 2} = 7 - x$ $-x^2 - 3x - 10 = 14 + 5x - x^2$ $8x = -24 \quad \text{CV } -3$ $x < -3 \quad -1 < x < 2$	<p>This sketch on its own scores no marks, but it may be seen in the work</p> <p>M1</p> <p>dM1</p> <p>A1A1</p> <p>M1</p> <p>A1</p> <p>dddM1A1A1</p> <p>[9]</p>
<p>NB</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>dddM1</p> <p>A1</p> <p>A1</p>	<p>No algebra implies no marks</p> <p>Form a quadratic equation or inequality, no simplification needed</p> <p>Solve the 3TQ any valid method Depends on the first M mark.</p> <p>Either CV</p> <p>Both CVs</p> <p>Change the sign of LHS or RHS and obtain an equation (quadratic or linear, no simplification needed)</p> <p>Correct CV from solving the linear equation</p> <p>$x <$ their smallest CV and x between their other 2 CVs All M marks above needed</p> <p>Either inequality correct</p> <p>Both inequalities correct</p> <p>“and” between the inequalities is acceptable. If \cap used, deduct an A mark.</p>	

Question Number	Scheme	Marks
4		
(a)	$ 18\sqrt{3} - 18i = 18\sqrt{(3+1)} = 36$ $\tan \theta = \frac{-18}{18\sqrt{3}} \quad \theta = -\frac{\pi}{6}, \quad 18\sqrt{3} - 18i = 36\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$	B1 M1, A1cao (3)
(b)	$z^4 = 36\left(\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6}\right) = 36\left(\cos\left(2k\pi - \frac{\pi}{6}\right) + i\sin\left(2k\pi - \frac{\pi}{6}\right)\right)$ $z = \sqrt[4]{36}\left(\cos\left(\frac{12k\pi - \pi}{24}\right) + i\sin\left(\frac{12k\pi - \pi}{24}\right)\right)$ $k = 0 \quad z_0 = \sqrt[4]{36}\left(\cos\left(\frac{-\pi}{24}\right) + i\sin\left(\frac{-\pi}{24}\right)\right) = \sqrt[4]{36} e^{i\left(-\frac{\pi}{24}\right)}$ $k = 1 \quad z_1 = \sqrt[4]{36}\left(\cos\left(\frac{11\pi}{24}\right) + i\sin\left(\frac{11\pi}{24}\right)\right) = \sqrt[4]{36} e^{i\frac{11\pi}{24}}$ $k = 2 \quad z_2 = \sqrt[4]{36}\left(\cos\left(\frac{23\pi}{24}\right) + i\sin\left(\frac{23\pi}{24}\right)\right) = \sqrt[4]{36} e^{i\frac{23\pi}{24}}$ $k = -1 \quad z_3 = \sqrt[4]{36}\left(\cos\left(-\frac{13\pi}{24}\right) + i\sin\left(-\frac{13\pi}{24}\right)\right) = \sqrt[4]{36} e^{i\left(-\frac{13\pi}{24}\right)}$	M1 M1 B1 A1ft A1ft (5) [8]
(a) B1 M1 A1cao (b) M1 M1 B1 A1ft A1ft NB	Correct modulus Attempt argument using $\tan \theta = \frac{\pm 18}{18\sqrt{3}}$ or other valid method. Can be implied by $\theta = \pm \frac{\pi}{6}$ Correct answer in the required form. Valid method for generating at least 2 roots, rotation through $\frac{\pi}{2}$ accepted Apply de Moivre or use the rotation method Any one correct root Second root in required form All 4 roots in the required form Follow through their $\sqrt[4]{36}$ but 36 not acceptable. Argument in degrees – M1M1B1A0A0 (ie treat as mis-read) Incorrect argument: B0A1ftA1ft available Answers in $r(\cos \theta + i\sin \theta)$ form – deduct final A marks	

Question Number	Scheme	Marks
5	$w = \frac{z - 3i}{z + 2i}$ $w(z + 2i) = z - 3i \quad z = \frac{i(2w + 3)}{1 - w}$ $ z = 1 \quad \left \frac{i(2w + 3)}{1 - w} \right = 1$ $ i(2w + 3) = 1 - w $ $w = u + iv \quad (2u + 3)^2 + 4v^2 = (1 - u)^2 + v^2$ $4u^2 + 12u + 9 + 4v^2 = 1 - 2u + u^2 + v^2$ $3u^2 + 3v^2 + 14u + 8 = 0$ $u^2 + v^2 + \frac{14}{3}u + \frac{8}{3} = 0$ $\left(u + \frac{7}{3}\right)^2 + v^2 = -\frac{8}{3} + \frac{49}{9} = \frac{25}{9}$ <p>(i) Centre $\left(-\frac{7}{3}, 0\right)$</p> <p>(ii) Radius $\frac{5}{3}$</p>	M1 dM1 ddM1 dddM1 A1 A1 A1 (7) [7]
(a) M1 dM1 ddM1 dddM1 A1 A1 A1	re-arrange to $z = \dots$ dep (on first M1) using $ z = 1$ with their previous result dep (on both previous M marks) use $w = u + iv$ (or any other pair of letters inc (x, y)) and find the moduli (or square of it) dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms) for a correct equation in u and v with coeffs of u^2 and v^2 both 1 Correct centre, must be in coordinate brackets. Completion of square need not be shown. Correct radius Centre and radius must come from a correct circle equation for the A marks	

Question Number	Scheme	Marks
6.	$\frac{dy}{dx} + \frac{(x \cot x + 2)}{x} y = \frac{4 \sin x}{x^2}$ $\text{IF} = e^{\int \frac{(x \cot x + 2)}{x} dx}$ $= e^{(\ln \sin x + 2 \ln x)}$ $= x^2 \sin x$ $\frac{d}{dx}(\text{their IF} \times y) = \text{their IF} \times \frac{4 \sin x}{x^2}$ $y x^2 \sin x = \int 4 \sin^2 x dx = 4 \int \frac{1 - \cos 2x}{2} dx = 4 \left(\frac{x}{2} - \frac{1}{4} \sin 2x \right) (+C)$ $y = \frac{2x - \sin 2x + C}{x^2 \sin x} \quad \text{oe}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>dM1A1</p> <p>A1cao [8]</p>
<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>A1</p>	<p>Divide through by x^2</p> <p>Attempt an IF of the form $e^{\int \frac{k(x \cot x + 2)}{x} dx}$</p> <p>$(\ln \sin x + 2 \ln x)$</p> <p>Correct IF</p> <p>Multiply through by their IF and write LHS in form shown – can be implied by next line. Allow if IF is seen instead of their function provided an IF has been attempted. Allow use of their RHS</p> <p>Attempt to integrate $\sin^2 x$, including using $\sin^2 x = \frac{1}{2}(1 \pm \cos 2x)$ $\cos 2x \rightarrow k \sin 2x$</p> <p>depends on previous M mark</p> <p>Correct integration, constant not needed</p> <p>Include the constant and treat it correctly. Must have $y = \dots$</p>	

Question Number	Scheme	Marks
7 (a)	$r \sin \theta = 2a \sin \theta + 2a \sin \theta \cos \theta \quad \text{OR} \quad r \sin \theta = 2a \sin \theta + a \sin 2\theta$ $\frac{d(r \sin \theta)}{d\theta} = 2a \cos \theta + 2a \cos^2 \theta - 2a \sin^2 \theta \quad \left \quad \frac{d(r \sin \theta)}{d\theta} = 2a \cos \theta + 2a \cos 2\theta \right.$ $2 \cos^2 \theta + \cos \theta - 1 = 0 \quad \text{terms in any order}$ $(2 \cos \theta - 1)(\cos \theta + 1) = 0$ $\cos \theta = \frac{1}{2} \quad \theta = \frac{\pi}{3} \quad (\theta = \pi \text{ need not be seen})$ $r = 2a \times \frac{3}{2} = 3a$	B1 M1 A1
(b)	$\text{Area} = \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4a^2 (1 + \cos \theta)^2 d\theta$ $= 2a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 + 2 \cos \theta + \cos^2 \theta) d\theta$ $= 2a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1 + 2 \cos \theta + \frac{1}{2} (\cos 2\theta + 1) \right) d\theta$ $= 2a^2 \left[\theta + 2 \sin \theta + \frac{1}{2} \left(\frac{1}{2} \sin 2\theta + \theta \right) \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= 2a^2 \left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} - \left(\frac{\pi}{6} + 1 + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{12} \right) \right]$ $= 2a^2 \left(\frac{\pi}{4} + \sqrt{3} - 1 \right)$ $\text{Area of } \triangle OAB = \frac{1}{2} \times 3a \times (2 + \sqrt{3})a \times \sin \frac{\pi}{6} \left(= \frac{3}{4} a^2 (2 + \sqrt{3}) \right)$ $\text{Shaded area} = 2a^2 \left(\frac{\pi}{4} + \sqrt{3} - 1 \right) - \frac{3}{4} a^2 (2 + \sqrt{3}) = \frac{a^2}{4} (2\pi - 14 + 5\sqrt{3})$	dM1A1 A1 (6) M1 M1 dM1A1 M1 NB: A1 on e-PEN M1A1cao (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	Multiply r by $\sin \theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen	
M1	Differentiate $r \sin \theta$ or $r \cos \theta$ (using product rule or using double angle formula first)	
A1	Correct derivative for $r \sin \theta$	
dM1	Use $\sin^2 \theta + \cos^2 \theta = 1$ to form a 3TQ in $\cos \theta$ and attempt its solution by a valid method	
A1	Correct value for θ	
A1	Correct r	
(b)		
M1	Use area $= \frac{1}{2} \int r^2 d\theta$ with $r = 2a + 2a \cos \theta$, no limits needed,	
M1	Use a double angle formula to obtain a function ready for integrating (Alt method uses integration by parts – may be seen)	
dM1	Attempt the integration $\cos 2\theta \rightarrow \frac{1}{k} \sin 2\theta$ $k = \pm 2$ or ± 1	
A1	Correct integration,	
M1	Substitute the limits (need not be simplified). Limits $\frac{\pi}{6}$ and their θ from (a) provided this is $> \frac{\pi}{6}$	
M1	NB: A1 on e-PEN Obtain the area of ΔOAB and subtract from their previous area	
A1	Correct answer	

Question Number	Scheme	Marks
8 (a)	$x = e^u \quad \frac{dx}{du} = e^u \quad \text{or} \quad \frac{du}{dx} = e^{-u} \quad \text{or} \quad \frac{dx}{du} = x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u} \frac{dy}{du}$ $\frac{d^2y}{dx^2} = -e^{-u} \frac{du}{dx} \frac{dy}{du} + e^{-u} \frac{d^2y}{du^2} \frac{du}{dx} = e^{-2u} \left(-\frac{dy}{du} + \frac{d^2y}{du^2} \right)$ $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 8y = 4 \ln x$ $e^{2u} \times e^{-2u} \left(-\frac{dy}{du} + \frac{d^2y}{du^2} \right) + 3e^u \times e^{-u} \frac{dy}{du} - 8y = 4 \ln(e^u)$ $\frac{d^2y}{du^2} + 2 \frac{dy}{du} - 8y = 4u \quad *$	<p>B1</p> <p>M1</p> <p>M1A1</p> <p>dM1</p> <p>A1*cso (6)</p>
<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>dM1 A1*cso</p>	<p>$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used</p> <p>Obtaining $\frac{dy}{dx}$ using chain rule here or seen later</p> <p>Obtaining $\frac{d^2y}{dx^2}$ using product rule (penalise lack of chain rule by the A mark)</p> <p>Correct expression for $\frac{d^2y}{dx^2}$ any equivalent form</p> <p>Substituting in the equation to eliminate x (u and y only). Depends on the 2nd M mark</p> <p>Obtaining the given result from completely correct work</p>	
	<p>ALTERNATIVE 1</p> $x = e^u \quad \frac{dx}{du} = e^u = x$ $\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du} = x \frac{dy}{dx}$ $\frac{d^2y}{du^2} = 1 \frac{dx}{du} \times \frac{dy}{dx} + x \frac{d^2y}{dx^2} \times \frac{dx}{du} = x \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2}$ $x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}$ $\left(\frac{d^2y}{du^2} - \frac{dy}{du} \right) + 3x \times \frac{1}{x} \frac{dy}{du} - 8y = 4 \ln(e^u)$ $\frac{d^2y}{du^2} + 2 \frac{dy}{du} - 8y = 4u \quad *$	<p>B1</p> <p>M1</p> <p>M1A1</p> <p>dM1A1*cso (6)</p>

Question Number	Scheme	Marks
B1	$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{dy}{du}$ using chain rule here or seen later	
M1	Obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1 A1*cso	Substituting in the equation to eliminate x (u and y only). Depends on the 2 nd M mark Obtaining the given result from completely correct work	
	ALTERNATIVE 2: $u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x} \frac{dy}{du}$ $\frac{d^2y}{dx^2} = -\frac{1}{x^2} \frac{dy}{du} + \frac{1}{x} \frac{d^2y}{du^2} \times \frac{du}{dx} = -\frac{1}{x^2} \frac{dy}{du} + \frac{1}{x^2} \frac{d^2y}{du^2}$ $x^2 \left(-\frac{1}{x^2} \frac{dy}{du} + \frac{1}{x^2} \frac{d^2y}{du^2} \right) + 3x \times \frac{1}{x} \frac{dy}{du} - 8y = 4u$ $\frac{d^2y}{du^2} + 2 \frac{dy}{du} - 8y = 4u \quad *$	B1 M1 M1A1 M1A1*cso
	Notes as for main scheme	

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x , y and u until the final stage.

Mark as follows:

B1	as shown in schemes above
M1	obtaining a first derivative with chain rule
M1	obtaining a second derivative with product rule
A1	correct second derivative with 2 or 3 variables present
dM1	Either substitute in equation I or substitute in equation II according to method chosen and obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso	Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2 + 2m - 8 = 0$ $(m + 4)(m - 2) = 0, \quad m = -4, 2$ $CF = Ae^{-4u} + Be^{2u}$ PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, $c = 0$) $\frac{dy}{du} = a \quad \frac{d^2y}{du^2} = 0$ $0 + 2a - 8(au + b) = 4u$ $a = -\frac{1}{2} \quad b = -\frac{1}{8}$ $\therefore y = Ae^{-4u} + Be^{2u} - \frac{1}{2}u - \frac{1}{8}$	M1A1 A1 M1 dM1A1 B1ft (7)
(c)	$y = Ax^{-4} + Bx^2 - \frac{1}{2}\ln x - \frac{1}{8}$	B1 (1) [14]
(b) M1 A1 A1 M1 dM1 A1 B1ft	Writing down the correct aux equation and solving to $m = \dots$ (usual rules) Correct solution ($m = -4, 2$) Correct CF – can use any (single) variable Using an appropriate PI and finding $\frac{dy}{du}$ and $\frac{d^2y}{du^2}$ Use of $y = \lambda u$ scores M0 Substitute in the equation to obtain values for the unknowns. Depends on the second M1 Correct unknowns two or three (with $c = 0$) A complete solution, follow through their CF and a non-zero PI. Must have $y = a$ function of u Allow recovery of incorrect variables.	
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x No ft here x^{-4} or $e^{-4\ln x}$ and x^2 or $e^{2\ln x}$ allowed. Must start $y = \dots$	